Patrician College of Arts and Science

Department of Mathematics

Statistics and Their Appllications II

SBAOD

Fourth Semester

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Binomial distribution :

- The Binomial distribution is all about success and failure

When to use the Binomial Distribution

- A fixed number of trials
- Only two outcomes (true, false; heads tails; girl,boy; six, not six …..)
- Each trial is independent

➢ If the random variable X has Binomial distribution, then we write **X~B(n,p)**

Binomial distribution:

If X ia a Random Variable the we have

$$P(m,N,p)=C_{N,m}p^m q^{N-m}=\binom{N}{m}p^m q^{N-m}=\frac{N!}{m!(N-m)!}p^m q^{N-m}$$

where p is probability of a success and

$q = 1 - p$ is probability of a failure

Sometimes you have to use the Binomial Formula

$$P(X=x)=\binom{n}{x}p^x \times q^{(n-x)},$$

$$where\; q = 1 - p$$

Mean and Variance of a discrete distribution

- Mean of binomial distribution:

$$\mu = np$$

- Variance of binomial distribution:

$\sigma^2 = npq$

- Standard Deviation of binomial distribution:

$\sigma = \sqrt{npq}$

where p+q=1

Poisson Distribution:

- Distribution often used to model the number of incidences of some characteristic in time or space:
  - Arrivals of customers in a queue
  - Numbers of flaws in a roll of fabric
  - Number of typos per page of text.
- Distribution obtained as follows:
  - Break down the "area" into many small "pieces" (*n* pieces)
  - Each "piece" can have only 0 or 1 occurrences (p=P(1))
  - Let $l=np \equiv$ Average number of occurrences over "area"
  - $Y \equiv$ # occurrences in "area" is sum of 0^s & 1^s over "pieces"
  - $Y \sim \text{Bin}(n,p)$ with $p = l/n$
  - Take limit of Binomial Distribution as $n \rightarrow \infty$ with $p = l/n$

- Formula :

$$P(X = x) = \frac{e^{-\lambda} \lambda^{y}}{y!}$$

where y=0,1,2,…

- Mean of Poisson distribution:

$$E(Y) = \lambda$$

- Variance of Poisson distribution:

$$E(Y^2) = \lambda$$

more on Poisson…

Poisson Process" (rates)

Note that the Poisson parameter λ can be given as the mean number of events that occur in a defined time period or equivalently, λ can be given as a rate, such as λ=2/month (2 events per 1 month) that must be multiplied by *t*=time (called a "Poisson Process") .

X ~ Poisson (λ)

$$P(X = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

E(X) = λt

Var(X) = λt

Normal Distribution

- A continuous Random Variable X is said to have a normal distribution with parameters

μ and σ, where $-\infty < \mu < \infty$ and $0 < \sigma$, if the pdf of X is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \qquad -\infty < x < \infty$$

Standard Normal Distributions

The normal distribution with parameter values
is called a *standard normal distribution*. The random variable is denoted by *Z*.

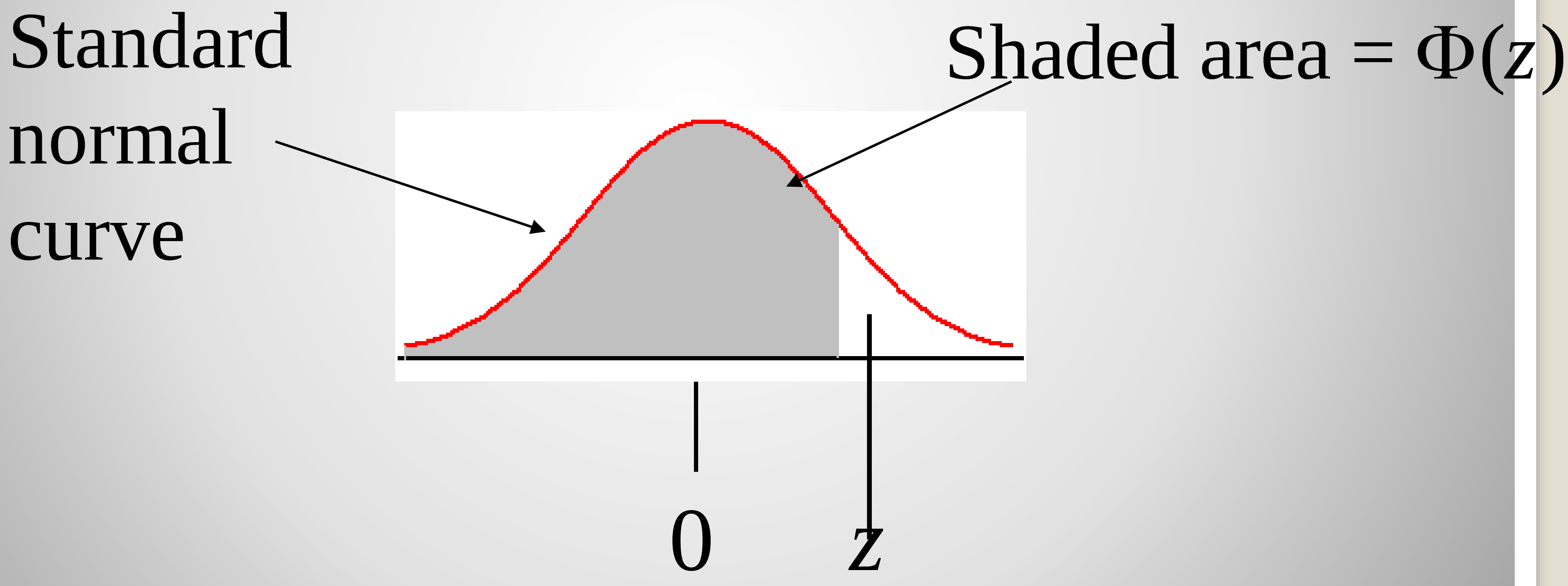
The pdf is

$$f(z;0,1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-z^2/2} \qquad -\infty < z < \infty$$

The cdf is

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} f(y;0,1)\,dy$$

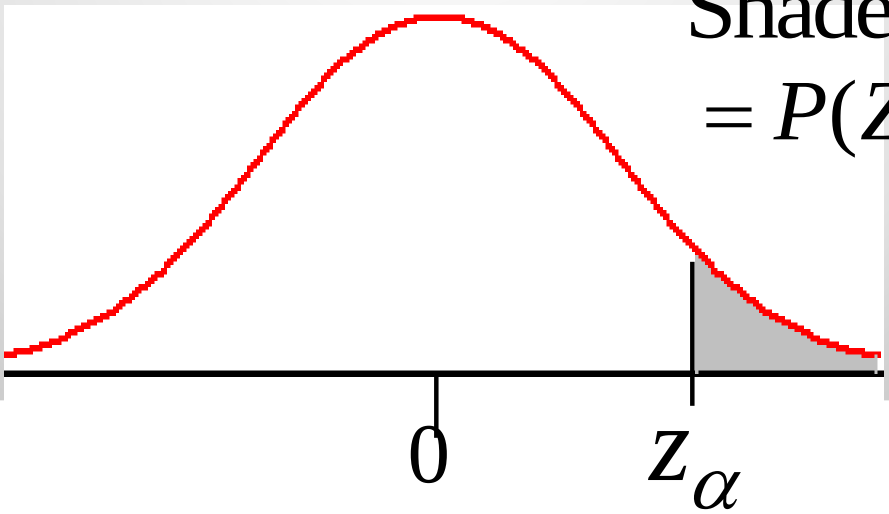
Standard Normal Cumulative Areas



z_α Notation

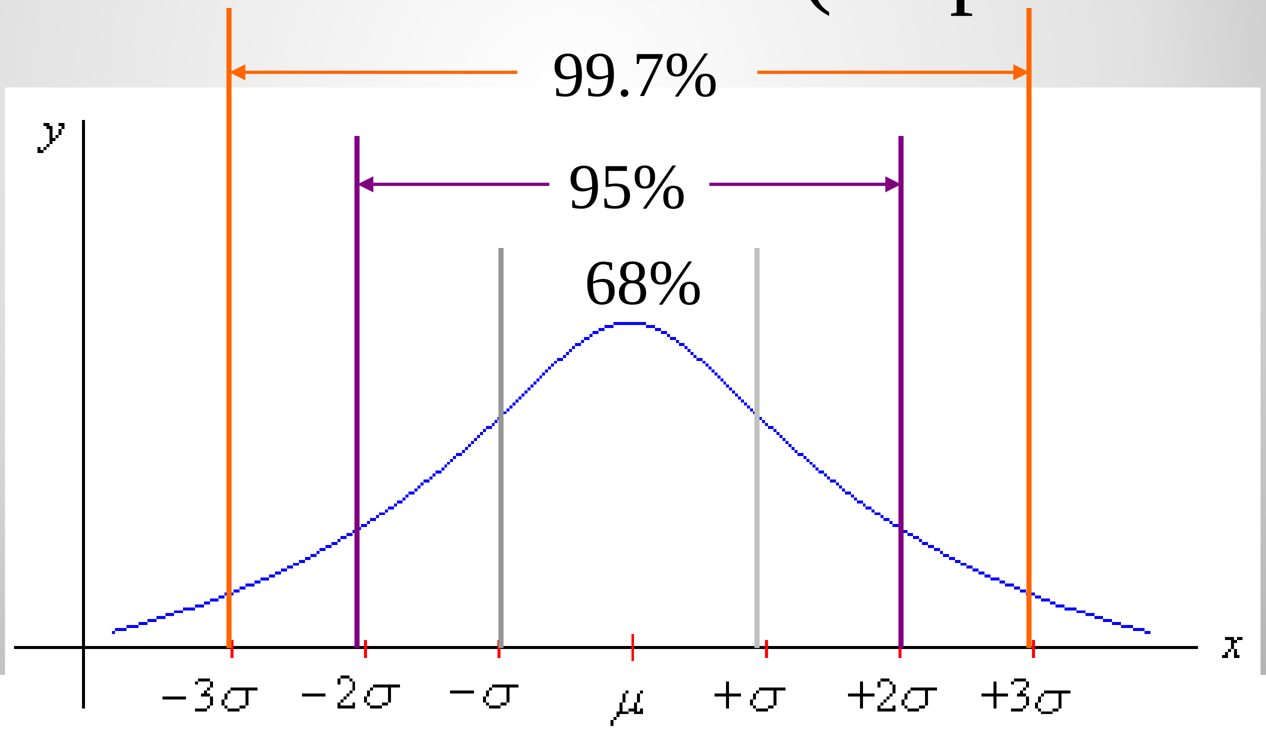
- z_α will denote the value on the measurement axis for which the area under the z curve lies to the right of z_α

Shaded area $= P(Z \geq z_\alpha) = \alpha$



Normal Curve

- Approximate percentage of area within given standard deviations (empirical rule).



Properties of Normal Distribution Curve

- The shape of the normal curve is often illustrated as a bell-shaped curve.
- The highest point on the normal curve is at the mean of the distribution.
- The normal curve is symmetric.
- The standard deviation determines the width of the curve.

- The total area under the curve the same as any other probability distribution is 1.
- The probability of the normal random variable assuming a specific value the same as any other continuous probability distribution is 0.
- Probabilities for the normal random variable are given by areas under the curve.

The Normal Probability Density Function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(x-\mu)^2/2\sigma^2}$$

where

μ = mean

σ = standard deviation

π = 3.14159

e = 2.71828

The Standard Normal Probability Density Function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

where

$\mu = 0$

$\sigma = 1$

$\pi = 3.14159$

$e = 2.71828$

THANK YOU